**Exponential Functions**

There is a connection between population growth, radioactive decay, musical scales,

and compound interest. They seem to have little in common, but you can model

any of them using an exponential function.

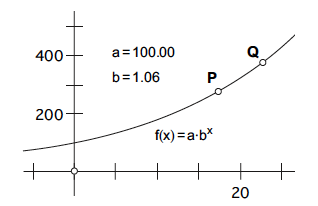
An exponential function has the general form *f*(*x*) = *ab*xwhere *a* ≠ 0, *b* >0, and

*b* ≠ 1.

DOUBLING PERIOD

Exponential functions can be used to solve a number of real-life problems. The function shows the value of $100 invested at an effective annual yield of 6%.

Look at the GSP file (page 1). The function shown has the definition *f*(*x*)= 100(1.06 )x. This shows the value of $100 invested at an effective annual yield of 6%. **(The *x* variable is in years, and *y* is in dollars.)**

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**Q1 After 10 years, how much money will you have? \_\_\_\_\_\_\_\_\_\_\_** (Drag point P to 10 years and look at the value for Yp)

**After 20 years, how much money will you have? \_\_\_\_\_\_\_\_\_\_\_**

In order to figure out how long will it take to double your money, first drag points P and Q so that the ratio yp / yp = is 2.00. Then, look at the difference in their *x*-coordinates?

**Q2 How long will it take to double your money? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**This number is called the *doubling period*.

**Q3 How long will it take to double your money?** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(Hint: Drag points P and Q so that the ratio is 4.0.)

Change the values if and b to show the function of $100 invested at an effective annual yield of 9%. (Hint: Double click on the value of b and change the value.)

**Q4 How long will it take to double your money? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5 How long will it take to quadruple your money? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

\*\*\* Do we want to have them make an observation about how the curve steepens as the interest rate increases??

HALF-LIFE

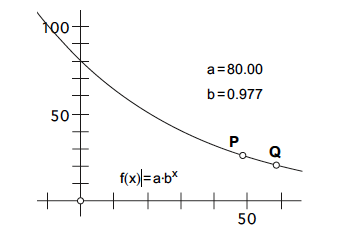
On the bottom of the GSP file, click on page 2.

An exponential function can also be used to model the decay of radioactive cesium.

10.

To model the decay of 80 g of cesium, the function is defined by *f*(*x*) = 80(0.977 )x

**The value of *x* is still in years, but the value of *y* is in grams.**



If you start with 80 g, you will have less cesium every year.

**Q6 How long would it take to lose half of it?** (Hint: Think about what value yp / yp would need to equal to show half and then move P and Q to reach that ratio. )

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ This number is called the *half-life* of cesium.

**Q7 Explain how you found the answer.**

Although cesium decays, as opposed to growing, you can still calculate its

doubling period. Drag the two points until you find a position where the ratio is

2.00.

**Q8 What is the difference in the *x*-coordinates?** Explain how this verifies your answer.